Geotechnical and Geophysical Site Characterisation 5 – Lehane, Acosta-Martínez & Kelly (Eds)
© 2016 Australian Geomechanics Society, Sydney, Australia, ISBN 978-0-9946261-2-7

Enhanced data interpretation: combining in-situ test data by Bayesian updating

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ABSTRACT: Combining data sets obtained during site characterization studies has the potential to enhance data interpretation. An example is provided where seismic data is combined with preconsolidation pressures obtained from laboratory tests. Semi-empirical relationships between preconsolidation pressure and shear wave velocity can be used to estimate the preconsolidation pressure. Conventionally, the uncertainties in the relationships are ignored. The Bayesian updating approach is used to take these uncertainties into account. Both trend analysis and kriging interpolation are used for the laboratory tests on the preconsolidation pressure to derive the prior distributions. The posterior preconsolidation pressures are then obtained by incorporating shear wave velocity measurements from a seismic dilatometer test. The analyses demonstrate that the seismic dilatometer data can be enhanced by the laboratory data set. This approach can be extended to enhance two-dimensional geophysical surveys by conditioning on a small number of accurate laboratory or in-situ test measurements.

1 INTRODUCTION

A typical site investigation program for a linear infrastructure project could include multiple types of site characterization techniques including geophysics, in-situ testing, borehole drilling and laboratory testing. Currently these data are often used in isolation from each other. For example, geophysical surveys could be used to assess site stratigraphy but not used to assess material parameters. When data are combined, they can have different resolutions. A profile of boreholes and in-situ tests can be overlain on geophysics data to help interpret stratigraphic boundaries from 'blurry' geophysics data. In-situ tests can be performed at different locations to boreholes and compared with the laboratory test data obtained from soil samples to develop material parameters. The soil between the in-situ tests and the borehole locations is assumed to have uniform properties.

Probabilistic numerical methods can be used to combine many of these different data sets to extract additional information from data that are routinely collected. The numerical methods have the potential to increase resolution of stratigraphic assessments using geophysics combined with other data, convert geophysics data to material properties, improve the resolution of in-situ tests combined with high quality laboratory measurements and to interpolate stratigraphy/material properties between test locations (e.g., Foti 2013).

Bayesian updating is a probabilistic numerical method that can be used to combine data. The advantage of Bayesian updating is that small data sets can be used. In this paper we demonstrate how Bayesian updating works by combining data from in-situ shear wave velocity measurements and laboratory tests with depth at a single location (one dimension). The theory can be extended to two and three dimensions. The example we present combines preconsolidation pressures obtained from constant rate of strain (CRS) consolidation tests with shear wave velocity data obtained from a seismic dilatometer test.

2 BAYESIAN UPDATING

Bayesian updating is a stochastic method that is well suited to geotechnical processes, particularly when limited information is available (e.g., Kelly and Huang 2015). Bayes' formula can be written as follows:

$$P(\boldsymbol{\theta}|\mathbf{y}) \propto P(\mathbf{y}|\boldsymbol{\theta})P(\boldsymbol{\theta}) \tag{1}$$

where $P(\boldsymbol{\theta})$ is the prior probability distribution of the material parameters, $P(\mathbf{y}|\boldsymbol{\theta})$ is the probability of measurements $(\mathbf{y})$ conditional on the material parameters $(\boldsymbol{\theta})$ and $P(\boldsymbol{\theta}|\mathbf{y})$ is the posterior distribution of the material parameters updated by measurements.

The measurements ($\mathbf{y}$) represent the geophysics data and can be written as:

$$y_i = f(\boldsymbol{\theta}) + \mu_\varepsilon \tag{2}$$

where μ_ε is the mean "error" or difference between measurement and model function (or calculation) $f(\boldsymbol{\theta})$.

If the measured error is assumed to be normally distributed, the likelihood function of measurement y_i can be written as:

$$P(y_i \mid \boldsymbol{\theta}) = \phi\left(\frac{y_i - f(\boldsymbol{\theta}) - \mu_\varepsilon}{\sigma_\varepsilon}\right) \tag{3}$$

where σ_ε is the standard deviation of the measurement errors, and ϕ is the probability density function of the standard normal distribution.

The Markov Chain Monte Carlo sampling method (MCMC) has been used to sample the posterior distribution. This method involves stepping through a Markov Chain where, at each step, a test realization for $\boldsymbol{\theta}$ is proposed according the proposed distribution, and is then either accepted or rejected using a random decision rule based on the realization's predicted data misfit and the misfit of the previously accepted model. After a certain "burn-in" period, required for the procedure to stabilize and become independent of the initial starting realization, accepted samples drawn at regular intervals along the Markov Chain will represent independent realizations of the posterior distribution and will occur at a frequency corresponding to their posterior probability of occurrence. The basic idea goes back to Metropolis et al. (1953)).

3 EMPIRICAL RELATIONSHIP BETWEEN PRECONSOLIDATION PRESSURES AND SHEAR WAVE VELOCITY

Atkinson (2007)) proposed Eq. (4) relating small strain shear stiffness to yield stress ratio. In this Equation, G_0 is the small strain stiffness, p_a is a normalising pressure taken to be 1kPa, A, n and m are constants, p' is the effective mean pressure and R_0 is the yield stress ratio.

$$\frac{G_0}{p_a} = A\left(\frac{p'}{p_a}\right)^n R_0^m \tag{4}$$

The shear wave velocity should therefore be related to the yield stress ratio because G_0 and shear wave velocity are related parameters. Eq. (5) is written in terms of effective vertical stress and OCR as an analogy to Equation 5.

$$\frac{G_0}{p_a} = A\left(\frac{\sigma'_v}{p_a}\right)^n OCR^m \tag{5}$$

A relationship between preconsolidation pressure and shear wave velocity was obtained by adopting the values n = 0.9, m = 0.35 and A = 170 estimated from Atkinson (2007)) and rewriting OCR in terms of preconsolidation pressure.

Eq. (5) is the model function and we consider that its accuracy is uncertain. The model function is used to convert the geophysical data into a form that can be compared with more accurate laboratory test data.

Preconsolidation pressures assessed from constant rate of compression tests and the ones interpreted from a seismic dilatometer test are compared in Fig. 1. The laboratory test data are considered the prior set of material parameters and the seismic dilatometer data are the measurements.

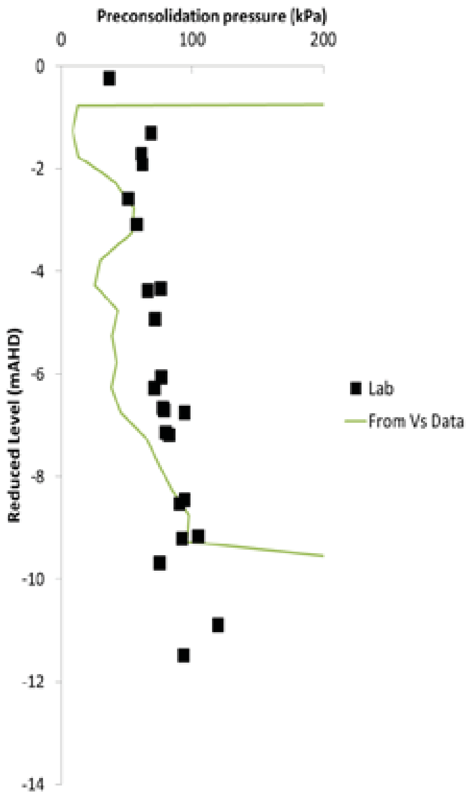

Figure. 1 Comparison between preconsolidation pressures assessed from constant rate of compression tests and interpreted from a seismic dilatometer test.

4 PRIOR INTERPRETATION OF PRECONSOLIDATION PRESSURES

The prior distribution is obtained from laboratory test data. A log-normal distribution has been assumed for the prior values.

A prior distribution of the measurements can be obtained by obtaining a linear trend through the data with depth and then assessing the standard deviation of the data from the trend. This is equivalent to assuming the preconsolidation pressure increases linearly with depth as

$$\sigma'_y = az + b \tag{6}$$

where z is reduced level, a and b are constants. The laboratory test data is fitted to Eq. (6) by the least square method, and $a = -6.38\text{kPa}$ and $b = 39.90\text{kPa}$. The standard deviation from the trendline is 6.39kPa and is assumed to be constant with depth.

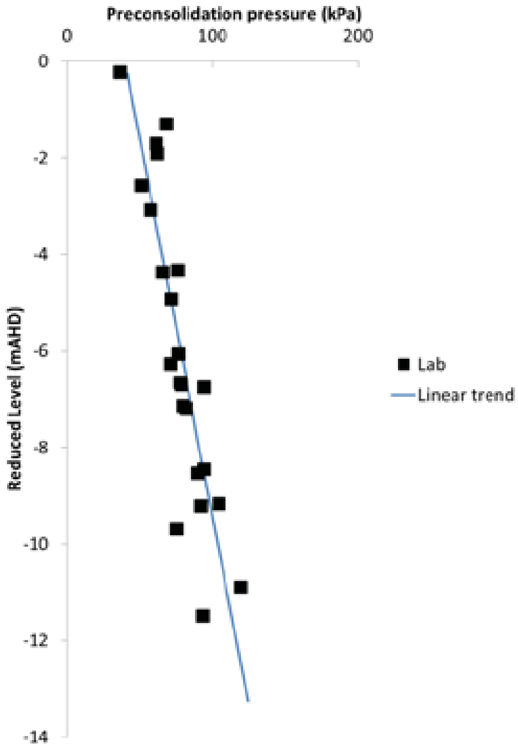

Figure. 2 Trend analysis of preconsolidation pressures assessed from constant rate of compression tests.

A prior distribution can also be obtained by kriging through the data set. Kriging provides a best estimate of a random field between known data. The basic idea is to estimate $X(z)$ at any point using a weighted linear combination of the values of X at each observation point. In kriging, high weights are given to the points that are closer to the unknown points. Interested readers are referred to Fenton and Griffiths (2008)). The kriged preconsolidation pressures assessed from constant rate of compression tests are shown in Fig. 3. In the example presented in this paper the scale of fluctuation is only used in the vertical direction but it can, in principle, be used to model 2D and 3D spatial variations.

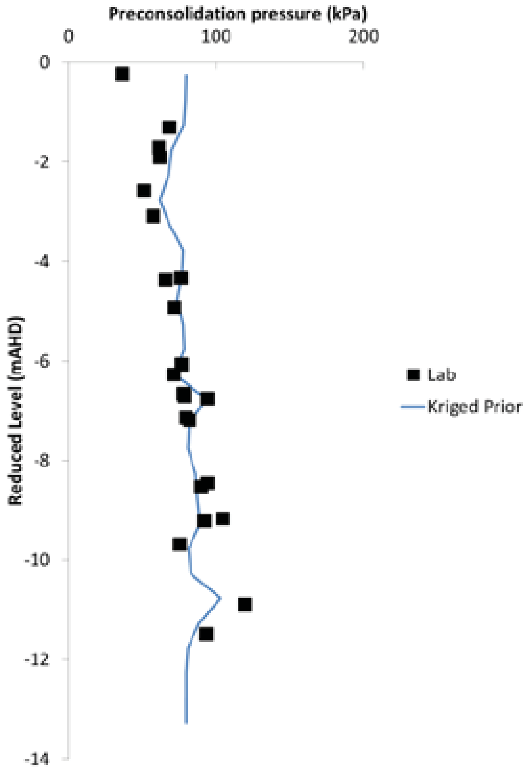

Figure. 3 Kriging interpolation of preconsolidation pressures assessed from constant rate of compression tests.

5 PRECONSOLIDATION PRESSURE UPDATED BY SHEAR WAVE VELOCITY

The mean and standard deviation of the error between the seismic dilatometer data and the model function are not known in advance. Shear wave velocities measured from seismic dilatometer tests are likely to be quite accurate, as is their conversion to small strain shear stiffnesses. However, the accuracy of the model function, Eq. (5), is quite uncertain. If we assume that there is no bias in the model function then the mean of the error can be set to zero. If we assume that the model function is highly uncertain then we can assign a large standard deviation to the error. In this case, the prior information will dominate the posterior solution, and in the extreme they will be equal. This is the same as assuming that the preconsolidation pressures measured in the laboratory apply everywhere in the domain. If we assume that the error has a small standard deviation then the solution will be more heavily influenced by the seismic dilatometer data and model function.

For the purpose of this example we have assumed that the standard deviation of the error is relatively small to highlight how the posterior is influenced by the prior and likelihood functions.

The posterior mean preconsolidation pressure is compared with the linear prior and model function from the seismic dilatometer in Fig. 4. The posterior prediction has updated the seismic dilatometer data such that its trend and magnitude more closely approximates the laboratory values.

A similar comparison with the kriged prior is shown in Fig. 5. Kriging incorporating a scale of fluctuation provides a set of prior values that are conditioned on the laboratory data but vary between data points according to the scale of fluctuation. The posterior prediction is a better fit to the laboratory test data, possibly due to the standard deviation of the prior kriged data being smaller than that of the prior linear data. Use of a scale of fluctuation with the kriged prior also provides greater accuracy when interpolating between data points with depth compared with the linear prior.

6 DISCUSSION

The previous section demonstrated some of the principles adopted when combining data. This method of analysis can be extended into two dimensions where a geophysical data set could be conditioned on laboratory and in-situ test data obtained at different spatial locations, both horizontally and with depth. The combined data set creates a 2D geotechnical model, where material parameters can be assigned to geophysical grid points. This model would be an enhancement of conventional models, where material parameters are assigned uniformly to a particular stratigraphic layer.

In principle, this model could then be transformed into a finite element mesh. Further, mean and standard deviation of the parameter values are also associated with the grid points, which allows creation of 2D probabilistic models. Creation of such models is a fundamental precursor for the routine use of probabilistic analysis in design practice.

However, a number of technical challenges remain to be overcome. One significant challenge is to quantify model error. At the moment it is difficult to quantify the magnitude of model error. Therefore the analyst can choose how much error to adopt in an analysis to effectively tune the model towards uniform material properties based on laboratory test data at one extreme or geophysics data at the other extreme. Selection of model error is therefore based on engineering judgment and the skill of the analyst.

A second challenge is to incorporate Bayesian updating of stratigraphic boundaries into the process (e.g., Houlsby and Houlsby 2013) before updating the material properties. In the examples presented in

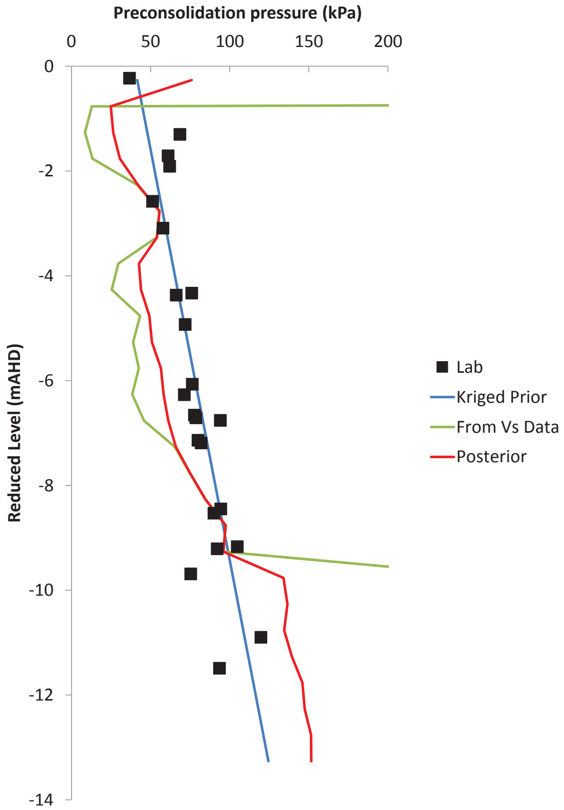

Figure. 4 Posterior mean preconsolidation pressures using trend analysis of constant rate of compression tests as prior distribution.

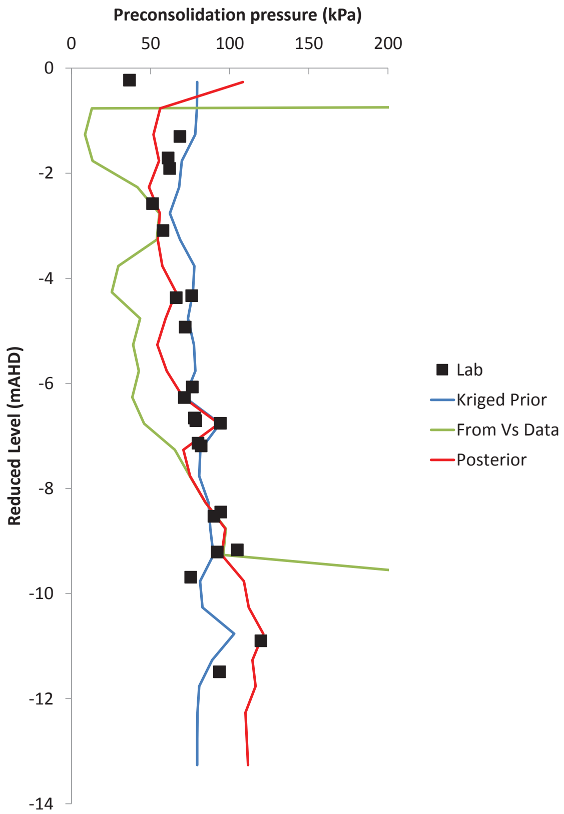

Figure. 5 Posterior mean preconsolidation pressures using kriging interpolation of constant rate of compression tests as prior distribution.

Section 5, the laboratory and dilatometer data sets were obtained from different physical locations and there is a difference in the depth to the base of the soft clay. The level at the base of the soft clay is approximately RL-9.5m at the location of the dilatometer and about RL-12m at the location of the laboratory test data. The Bayesian process, as presented here, updates both data sets irrespective of their stratigraphy.

7 CONCLUSIONS

This paper combines two different sets of geotechnical test data, namely results from laboratory constant rate compression tests and field seismic dilatometer tests. Empirical relationships between the preconsolidation pressure and shear wave velocity are firstly derived from the two sets of data. Unlike the traditional direct transformation, where the uncertainty associated with the transformation is ignored, the Bayesian updating approach is used to form a rigorous framework for combining the two sets of data. It is shown that the uncertainties of preconsolidation pressure can be significantly reduced by incorporating shear wave velocity measurements.

Further work is required to extend the process into two dimensions, incorporate stratigraphic updating and to investigate the magnitude of model error.

8 ACKNOWLEDGEMENTS

The authors wish to acknowledge the support of the Australian Research Council Centre of Excellence for Geotechnical Science and Engineering.

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